

DOCUMENT RESUME

ED 281 753

SE 047 947

AUTHOR Knoth, Russell L.; Benassi, Victor A.
 TITLE The Conjunction Error in Subjective Probability Assessment.
 PUB DATE Apr 87
 NOTE 12p.; Paper presented at the Annual Meeting of the Western Psychological Association (67th, Long Beach, CA, April, 1987).
 PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
 EDRS PRICE MF01/PC01 Plus Postage.
 DESCRIPTORS *College Mathematics; Educational Research; *Error Patterns; Higher Education; *Mathematical Concepts; *Probability; *Problem Solving
 IDENTIFIERS *Mathematics Education Research

ABSTRACT

The purpose of the study was to determine whether students had knowledge of the extension rule (the conjunction of two or more events cannot be greater than the probability of any one of those events) and understanding of conjunction by giving a test of multiplicative probabilities. Involved were 598 students enrolled in introductory psychology courses. A 10-item test of mathematical skills was given; it contained simple, additive, and multiplicative probability problems. Students were assigned randomly to either an instructional set condition or no instructional set condition. It was found that the majority of students were influenced by the representativeness heuristic. Overall, 83% of the students failed to use the extension rule to evaluate the conjunction problem. The vast majority (85%) did not answer either of the multiplicative problems correctly, thus indicating a failure to calculate objective compound probabilities. (MNS)

 * Reproductions supplied by EDRS are the best that can be made *
 * from the original document. *

ED281753

The Conjunction Error in Subjective Probability Assessment

Russell L. Knoth and Victor A. Benassi

University of New Hampshire

Paper presented at Western Psychological Association

67th Annual Meeting

Long Beach, California

April, 1987

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)
☒ This document has been reproduced as
received from the person or organization
originating it.
☐ Minor changes have been made to improve
reproduction quality.
• Points of view or opinions stated in this docu-
ment do not necessarily represent official
OERI position or policy.

"PERMISSION TO REPRODUCE THIS
MATERIAL HAS BEEN GRANTED BY

Russell L. Knoth
Victor A. Benassi

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)."

BEST COPY AVAILABLE

SE 0417 947

The Conjunction Error in Subjective Probability Assessment

Kahneman and Tversky (1972) described several heuristics that people rely on in intuitive judgment. One such heuristic, representativeness, is defined as the tendency of people to evaluate the probability of an event by the degree to which the event is "similar in essential properties to its parent population" and the degree to which it "reflects the salient process by which it was generated" (p. 431). Kahneman and Tversky suggested that subjects rate an event's probability higher when it is similar in nature to other events and/or when it appears to be generated by the other events. However, an event's objective probability is not affected by the same factors that affect its representativeness. Although probability and representativeness are often correlated, subjects tend to overestimate this correlation (Kahneman & Tversky, 1972). This overestimation, in turn, leads to the errors described.

One area in which people's subjective probability assessments do not match objective probabilities is in the evaluation of compound events. In such cases, subjects are asked to estimate the probabilities of various attributes or combinations of attributes. A typical problem is presented below (from Tversky & Kahneman, 1982):

Bill is 34 years old. He is intelligent, but unimaginative, compulsive, and generally lifeless. In school, he was strong in mathematics but weak in social studies and humanities.

Subjects are asked to rank order a number of events by their likelihood. For example:

Bill is an architect.

Bill is an accountant

Bill plays jazz for a hobby.

Bill is an accountant who plays jazz for a hobby.

The conjunction of two or more events cannot be greater than the probability of any one of those events (this is called the extension rule in probability calculus). In the example above, the probability that Bill is both an accountant and plays jazz for a hobby can not be greater than that of Bill just playing jazz, i.e., $p(A \text{ intersect } B) \leq p(A)$, or $p(B)$. However, Tversky and Kahneman (1982) demonstrated that the vast majority of subjects choose the conjunction statement as more probable than the single element statement, thus committing the "conjunction error" (Morier & Brogida, 1984). That is, most subjects ranked the statement that Bill was both an accountant and played jazz for a hobby as more probable than the single statement that Bill only played jazz because, according to Tversky and Kahneman, the conjunction statement is more representative than the single element statement.

The failure of subjects to utilize the extension rule when solving conjunction problems could be due to several reasons. First, it is possible that the simple appearance of the problem does not lead subjects to understand that the problem is mathematical or probabilistic in nature. That is, when subjects see the instruction "Choose the statement that is most probable" they do not think that they are being asked to generate a probability in a statistical sense; they may think they are being asked for a likely opinion. This assumes that subjects have the extension rule in their problem solving repertoires. The conjunction error, then, would be a failure to apply

the rule to this type of problem. One way to allievate this problem would be an instructional set directing subjects that the problem was statistical in nature.

A second reason for the overwhelming incidence of conjunction error might be the absence of the extension rule from many subjects' problem solving repertoires. A way to test this possibility would be to assess subjects' understanding of conjunction by an objective test of multiplicative probabilities. Though a strict measure of knowledge of the extension rule, it follows that if subjects can demonstrate such knowledge with objective probabilities they could apply it to conjunction problems, especially once so instructed.

Method

Subjects. Five hundred ninety-eight students enrolled in introductory psychology courses at the University of New Hampshire participated as part of a course requirement. Subjects were unselected for sex.

Materials. A ten item test of mathematical skills was given to all subjects. The test contained simple, additive, and multiplicative probability problems. Two multiplicative probability problems required logic identical to that necessary to solve the conjunction problem:

1. What is the probability of flipping a coin twice and getting heads on the first toss and tails on the second?
2. What is the probability of rolling a die three times and getting six every time?

Subjects' performance on these two problems was used to partition them into three groups: high, moderate, and low knowledge of multiplicative probability.

The following conjunction problem was used:

Dan is 28 years old. He is outgoing and well liked. He enjoys working with his hands. He has had many girlfriends but only one serious relationship.

Concerning the above description, please choose the statement that is most probable:

1. Dan is a carpenter who surfs for a hobby.
2. Dan surfs for a hobby.

Procedure. After receiving introductory information, subjects were randomly assigned to either an instructional set condition or a no instructional set condition. In the instructional set condition subjects read the following instructions prior to the completion of the conjunction problem:

To assist you in choosing the best answer to the problem you are about to read, I would like to emphasize that it would be helpful if you were to consider the following problem in mathematical terms. That is, the application of a simple mathematical rule will lead you to choose the correct answer.

In the no instructional set condition, the same procedure described above was used with the exception of the instructional set.

After subjects completed the problem, they were asked to write a justification for their answer. That is, they were asked to explain why they chose their answer as more probable. After completing the justification, they were given the 10 item mathematical skills test.

Results

Previous research by the authors (Knoth & Benassi, 1987) found that the most accurate measure to assess the degree to which subjects base their answers on representativeness, the conjunction rule, or some other logic when solving a conjunction problem is the justification for their answer. Judgments were rated as being based on the representativeness heuristic when they stated that some characteristic of the statement was similar or dissimilar to the paragraph. For example, the justification "Dan is a carpenter because he enjoys working with his hands" was rated as based on representativeness. On the other hand, the justification "The chances that Dan surfs and is a carpenter are less than he only surfs" was rated as based on the conjunction rule. All other justifications were put in the "Other" category. Judgments were then rated as correct when they were based on the extension rule and incorrect if they were not.

The two multiplicative probability problems described above were used to partition subjects into three groups. Subjects were placed in the low knowledge of multiplicative probability group if they got neither of the problems correct, in the moderate knowledge group if they got one of the two problems correct, and in the high knowledge group if they got both correct. The proportion of subjects in each of

these groups, from low to high knowledge, was 85%, 8%, and 7%.

The frequency and proportion of subjects, who got the Dan problem correct are presented in Table 1. Collapsing across all cells, 83% of all subjects committed the conjunction error.

A 2 (instruction/no instruction) X 3 (low/moderate/high knowledge) analysis of variance (ANOVA) revealed the following results. There was a nonsignificant main effect for the instructional set manipulation, $F(1,597) = 2.16, p = .14$. The proportions of correct answers in the instructional set and no instructional set groups were .19 and .14, respectively. There was a significant main effect for prior knowledge of multiplicative probability, $F(2,597) = 6.28, p = .002$. The proportion of subjects answering the problem correctly in the low, moderate, and high knowledge groups was .15, .16, and .38, respectively. These results indicate that prior knowledge of multiplicative probability can influence subjects' tendencies to evaluate the conjunction problem by the extension rule. The instructional set X knowledge interaction was nonsignificant, $F < 1$.

Although the interaction effect did not reach a standard level of significance, one planned comparison was tested. It was expected that subjects in the high knowledge group who were given the instructional set to evaluate the problem mathematically would be the least likely to commit the conjunction error. As Table 4 indicates, subjects in this group were more likely than all other groups to correctly utilize the extension rule to evaluate the probability of the statements. When the proportion of subjects utilizing the extension rule in this

group is compared to the proportion of subjects utilizing the extension rule in all other groups, the results indicate that the high knowledge group was significantly less likely to commit the conjunction error, $\chi^2 (1) = 13.39, p < .05$.

Discussion

The results of this experiment demonstrate that in conjunction problem evaluation, the majority of subjects are influenced by the representativeness heuristic. Overall, 83% of the subjects failed to use the extension rule to evaluate the conjunction problem. This result is comparable to the 88% found by Tversky and Kahneman (1982) using a different conjunction problem.

The results also indicate that the vast majority of subjects (85%) did not answer either of the multiplicative problems correctly, thus indicating a failure to calculate objective compound probabilities. The small proportion of subjects in the high knowledge group were most likely to utilize the extension rule in the evaluation of the Dan problem, although the percentage of subjects in this group who did so was still low. Therefore, when subjects' ability to calculate an objective probability is used as the indicator of an implicit understanding of the extension rule, only those subjects with such an understanding are able to use this rule to evaluate the conjunction problem.

The ability to calculate an objective compound probability is a strict criterion on which to base the assumption of an implicit understanding of conjunctive probabilities. However, other studies

performed by the authors (Knoth & Benassi, 1987) have shown similar results when more subjective probability evaluations (cf. Yates & Carlson, 1986) than examined in the present experiment are used to partition subjects. In these studies, the most likely subjects to use the extension rule to evaluate a conjunction problem are those who show knowledge of the extension rule. For this small percentage of people, the conjunction error appears to be a failure to apply the rule. For the rest, the conjunction error may be due to little or no knowledge of the rule.

References

- Kahneman, D., & Tversky, A. (1972). Subjective probability: A judgment of representativeness. Cognitive Psychology, 3, 430-454.
- Knoth, R. L., & Benassi, V. B. (1987). Extensional reasoning in the evaluation of compound events: Attempts to reduce the conjunction error. Manuscript in preparation.
- Morier, D. M., & Borgida, E. (1984). The conjunction fallacy: A task specific phenomenon? Personality and Social Psychology Bulletin, 10, 243-252.
- Tversky, A., & Kahneman, D. (1982). Judgments of and by representativeness. In D. Kahneman, P. Slovic, & A. Tversky (Eds.), Judgments Under Uncertainty: Heuristics and Biases. New York: Cambridge University Press.
- Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. Psychological Review, 90, 293-315.
- Yates, F. J., & Carlson, B. W. (1986). Conjunction errors: Evidence for multiple judgment procedures, including "Signed Summation". Organizational Behavior and Human Decision Processes, in press.

Table 1

Proportion and number of subjects by condition who answered the Dan problem correctly (Number of subjects in each group presented in parentheses).

Condition	Knowledge		
	low	moderate	high
No Instruction	.14 (252)	.11 (27)	.27 (15)
Instruction	.17 (257)	.23 (22)	.44 (25)